

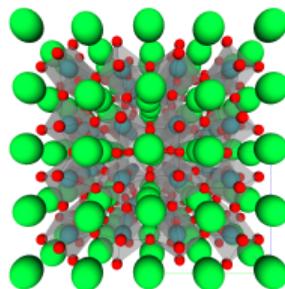


LIÈGE université
CESAM



Abinit School Prague 2019

The Multibinit project



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CESAM / Q-MAT

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Who is Multibinit ?

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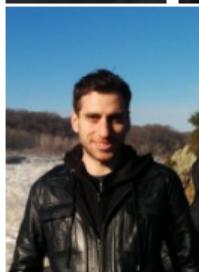
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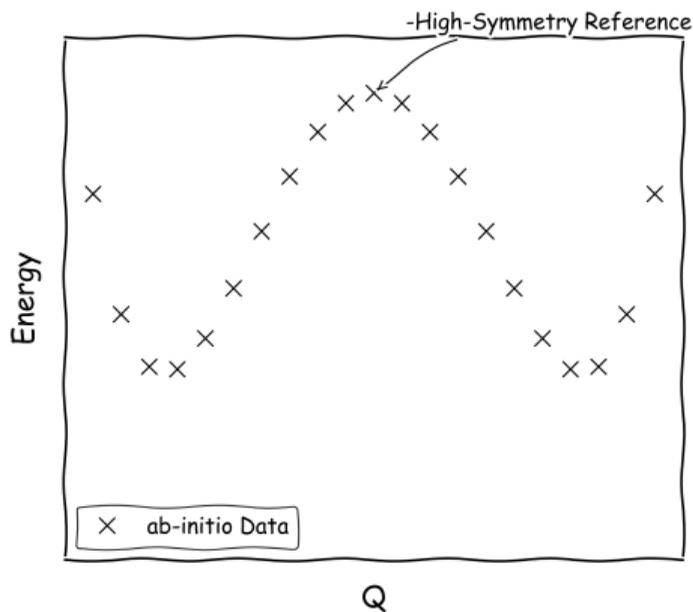
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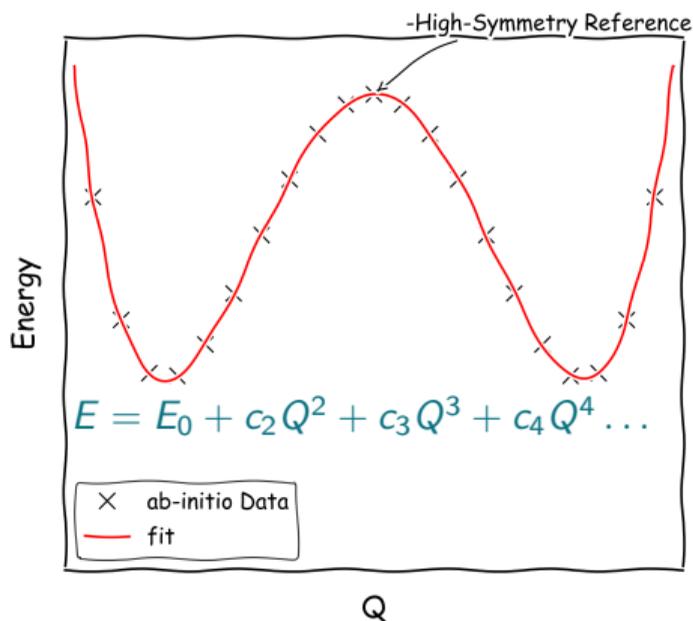


Transform Ab-Initio Data into Polynomial Description



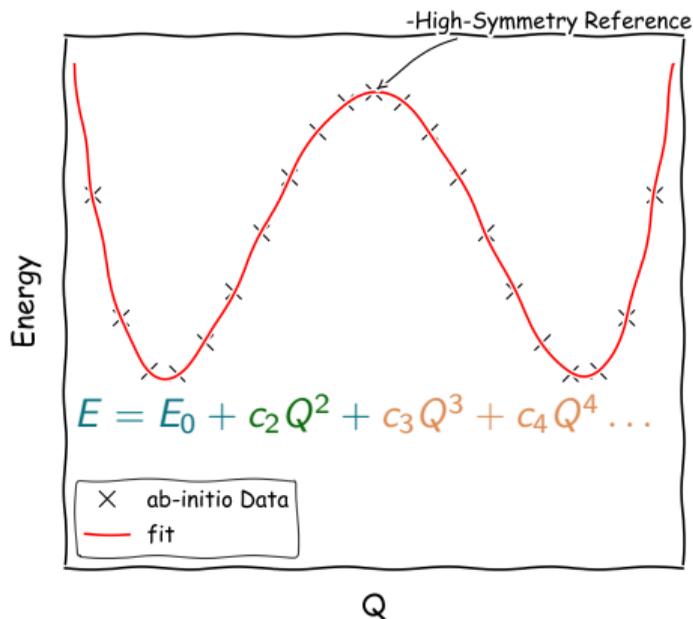
Q: A direction of displacements (\mathbf{u}, η)

Transform Ab-Initio Data into Polynomial Description



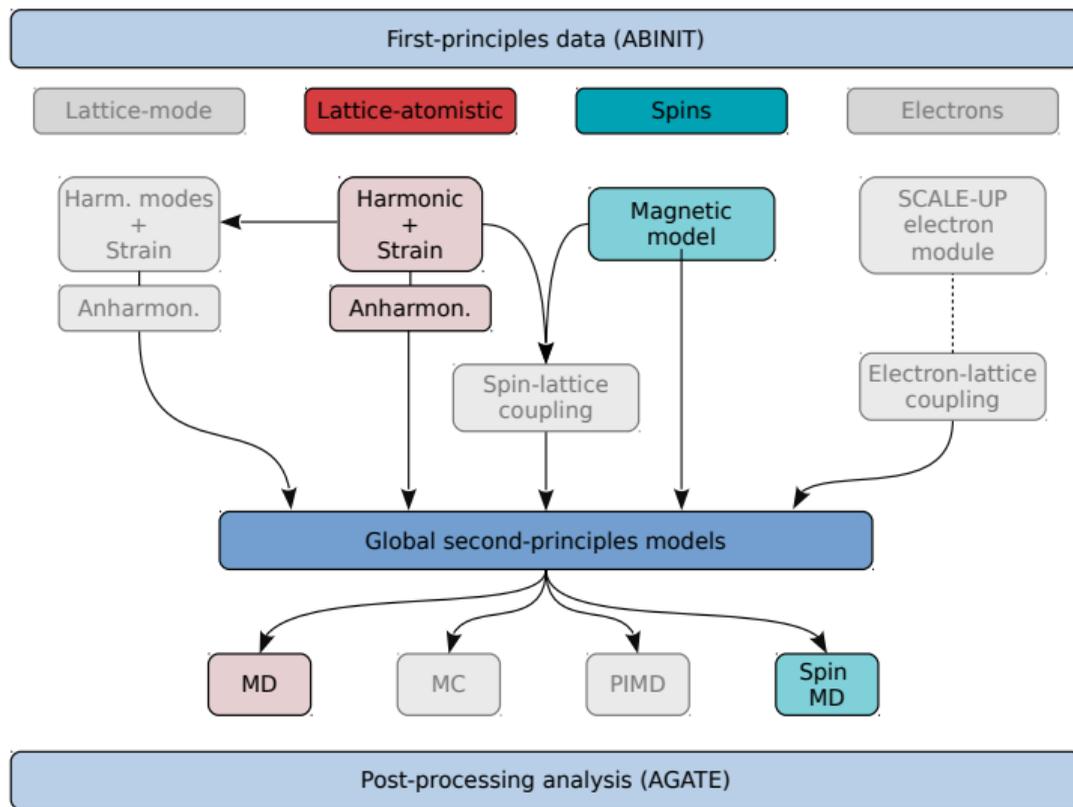
Q: A direction of displacements (\mathbf{u}, η)

Transform Ab-Initio Data into Polynomial Description



Q: A direction of displacements (\mathbf{u}, η)

Harmonic part extracted from DFPT - Higher order Fitted



Multibinit in the package

- Multibinit is a new executable included in the main directory:
src/98_main/multibinit
- Source to be found in *src/78_effpot*
- New input have been created (mix between anaddb and ABINIT)
- XML files are used for storing model informations (Fast reading)
- MPI Parallelization
- Abinit movers for Molecular Dynamics
- Documentation with tutorials and topics on Abinit Website (Work in progress..!)



Basic Procedure

- Express energy as low order Taylor expansion
 - Use high-symmetry reference structure
 - Variables: selected local modes and strains
- Determine coefficients from a limited number of DFT calculations
- Resulting energy expansion allows for finite temperature Monte Carlo

Successful results already for many oxide systems.

However identifying a small set of degrees of freedom is often difficult.

We need a general description of the energetics of all the atomic degrees of freedom

K. Rabe and J. Joannopoulos, PRL 59, 570 (1987); PRB 36, 6631 (1987)

W. Zhong et al., PRL 73, 1861 (1994); PRB 52, 6301 (1995)

Energy changes around reference structure due to distortions

$$E_{tot}(\{\mathbf{u}_i\}, \eta) = E_0(\{\mathbf{r}_0\}, 0) + E_{eff}(\{\mathbf{u}_i\}, \eta)$$

$$E_{eff}(\{\mathbf{u}_i\}, \eta) = E_p(\{\mathbf{u}_i\}) + E_s(\eta) + E_{sp}(\{\mathbf{u}_i\}, \eta)$$

Energy changes around reference structure due to distortions

$$E_{tot}(\{\mathbf{u}_i\}, \eta) = E_0(\{\mathbf{r}_0\}, 0) + E_{eff}(\{\mathbf{u}_i\}, \eta)$$

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Energy change due to
strain only

$$E_s^{Harm}(\eta)$$

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Energy change from
atomic displacements

Energy change due to
strain only

$$E_p^{Harm}(\{\mathbf{u}_i\}) + E_p^{Anharm}(\{\mathbf{u}_i\})$$

$$E_s^{Harm}(\eta)$$

Energy changes around reference structure due to distortions

$$E_{tot}(\{\mathbf{u}_i\}, \eta) = E_0(\{\mathbf{r}_0\}, 0) + E_{eff}(\{\mathbf{u}_i\}, \eta)$$

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Energy change from
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$$E_p^{Harm}(\{\mathbf{u}_i\}) + E_p^{Anharm}(\{\mathbf{u}_i\})$$

$$E_s^{Harm}(\eta)$$

$$E_{p,SR}^{Harm}(\{\mathbf{u}_i\}) + E_{p,LR}^{Harm}(\{\mathbf{u}_i\})$$

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Energy change from
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Strain-phonon cou-
pling

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$$E_p^{Harm}(\{\mathbf{u}_i\}) + E_p^{Anharm}(\{\mathbf{u}_i\})$$

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$$E_{sp}^{Harm}(\{\mathbf{u}_i\}, \eta) + E_{sp}^{Anharm}(\{\mathbf{u}_i, \eta\})$$

$$E_{p,SR}^{Harm}(\{\mathbf{u}_i\}) + E_{p,LR}^{Harm}(\{\mathbf{u}_i\})$$

Same framework as H_{eff} but includes all-atomic degrees of freedom.

J. Wojdel et al., J. Phys. Condens. Matter 25 (2013) 305401

$$\text{Phonon term } E_p(\{\mathbf{u}_i\}) = E_p^{\text{Harm}}(\{\mathbf{u}_i\}) + E_p^{\text{Anharm}}(\{\mathbf{u}_i\})$$

Taylor development around the reference structure

$$E_p(\{\mathbf{u}_i\}) = \underbrace{\frac{1}{2} \sum_{i\alpha j\beta} K_{i\alpha j\beta}^{(2)} u_{i\alpha} u_{j\beta}}_{\text{Harmonic terms}} + \underbrace{\frac{1}{6} \sum_{i\alpha j\beta k\gamma} K_{i\alpha j\beta k\gamma}^{(3)} u_{i\alpha} u_{j\beta} u_{k\gamma}}_{\text{Anharmonic terms}} + \mathcal{O}(u^4)$$

with $K_{i\alpha j\beta\dots}^n = \left. \frac{\partial^n E_{\text{eff}}}{\partial u_{i\alpha} \partial u_{j\beta} \dots} \right|_{\eta=0}$.

- Must comply the Acoustic Sum Rule \rightarrow difficult to enforce for $n > 2$
- Use displacement differences \rightarrow ASR satisfied by construction

$$E_{\text{Harm}}(\{\mathbf{u}_i\}) = \frac{1}{2} \sum_{\substack{i,j,k,h \\ \alpha,\beta}} \tilde{K}_{ij\alpha kh\beta}^{(2)} (u_{i\alpha} - u_{j\alpha})(u_{k\beta} - u_{h\beta})$$

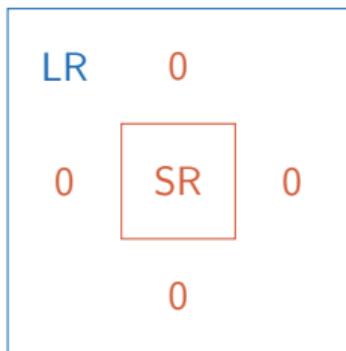
$$E_p^{Harm}(\{\mathbf{u}_i\}) = E_{p,SR}^{Harm}(\{\mathbf{u}_i\}) + E_{p,LR}^{Harm}(\{\mathbf{u}_i\})$$

$$\begin{aligned} E_p^{Harm}(\{\mathbf{u}_i\}) &= \frac{1}{2} \sum_{i\alpha j\beta} K_{i\alpha j\beta}^{(2)} u_{i\alpha} u_{j\beta} = \frac{1}{2} \sum_{i\alpha j\beta} (S_{i\alpha j\beta} + L_{i\alpha j\beta}) u_{i\alpha} u_{j\beta} \\ &= \frac{1}{2} \sum_{\substack{i \in SC \\ \alpha}} u_{i\alpha} \sum_{\substack{j \in SR \\ \beta}} S_{i\alpha j\beta} u_{j\beta} + \frac{1}{2} \sum_{\substack{i \in SC \\ \alpha}} u_{i\alpha} \sum_{\substack{j \in SC \\ \beta}} \underbrace{\left(\sum_b^{\infty} L_{i\alpha j\beta}(b) \right)}_{\tilde{L}_{i\alpha j\beta}(\mathbf{q}=0)} \underbrace{u_{j\beta}^b}_{u_{j\beta}^0} \end{aligned}$$

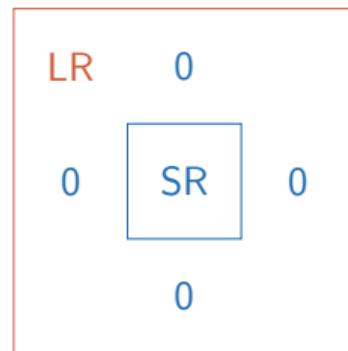
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SR < LR



LR < SR





$$E_p^{Anharm}(\{\mathbf{u}_i\})$$

- Anharmonicity degree : chose from maximum order: 3, 4, 5, 6, ...
- For each order build the possible terms

$$E_p^{(n)}(\{\mathbf{u}_i\}) = \frac{1}{n!} \sum_{\substack{i,j,k,l,m,n,\dots \\ \alpha,\beta,\gamma,\dots}} K_{ijklmn}^{(n)\alpha\beta\gamma} (u_{i\alpha} - u_{j\alpha})(u_{k\beta} - u_{l\beta})(u_{m\gamma} - u_{n\gamma}) \dots$$

- ASR enforces by using differences
- Use symmetries to reduce de number of coefficients

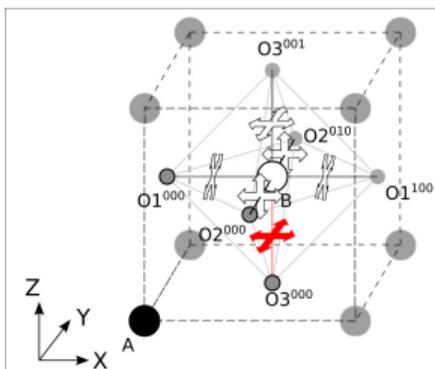


$$E_p^{Anharm}(\{\mathbf{u}_i\})$$

Symmetry considerations: "Symmetry adapted terms"

The operations of the space group can generate a collection of symmetry related terms which reduces the number of *independent* parameters. In practice :

- Define range or cutoff radius
- Build all terms within this range
- Apply all symmetries to find relations between terms :



$(u_{0Bx} - u_{0O3x})^2(u_{0By} - u_{0O3y})^2 + \frac{\pi}{2}$ rotation about the y-axis transforms to $(u_{0Bz} - u_{0O1z})^2(u_{0By} - u_{0O1y})^2$
 \rightarrow **15 independent parameters**
 (1st nearest neighbors at 3rd and 4th order in u) for the anharmonic phonon part.

$$\text{Phonon term } E_p(\{\mathbf{u}_i\}) = E_p^{\text{Harm}}(\{\mathbf{u}_i\}) + E_p^{\text{Anharm}}(\{\mathbf{u}_i\})$$

Harmonic terms

- $K^{(2)}$ obtained by DFPT as the back-Fourier-transformed dynamical matrix
 - Remove dipole-dipole interaction in real space to obtain the SR only.
 - Use ewald summation to compute the exact dipole-dipole interaction.
- ⇒ Harmonic terms exact by construction and include all phonon branches

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Anharmonic terms

- Displacement differences expression allows for an unconstrained optimization of $\tilde{K}^{(n)}$
- Determined by fitting key quantities to limited number of DFT calculations (training set)

Strain term $E_s(\eta)$

Taylor development around the reference structure

$$E_s(\eta) = \frac{N}{2} \sum_{ab} C_{ab}^{(2)} \eta_a \eta_b + \frac{N}{6} \sum_{abc} C_{abc}^{(3)} \eta_a \eta_b \eta_c + \mathcal{O}(\eta^4)$$

with $C_{ab\dots}^{(m)} = \frac{1}{N} \frac{\partial^m E_{\text{eff}}}{\partial \eta_a \partial \eta_b \dots} \Big|_{u_i=0}$

- Harmonic term calculated directly : the frozen ion elastic constants (DFPT)
- Anharmonic terms : in practice not required for semi-quantitative results

J. Wojdel et al., J. Phys. Condens. Matter 25 (2013) 305401

Strain-Phonon coupling $E_{sp}(\{u_i\}, \eta)$

Taylor development also subject to ASR

$$E_{sp}(\{u_i\}, \eta) = \underbrace{\frac{1}{2} \sum_a \sum_{i\alpha} \Lambda_{ai\alpha}^{(1,1)} \eta_a u_{i\alpha}}_{\text{Forces at homogeneous strain}} + \underbrace{\frac{1}{6} \sum_a \sum_{i\alpha j\beta} \Lambda_{ai\alpha j\beta}^{(1,2)} \eta_a u_{i\alpha} u_{j\beta}}_{\text{Change in force csts with strain}} + \dots$$

- The $\Lambda^{(m,n)}$ must comply with a set of ASRs
- Use displacement differences

$$E_{sp}(\{u_i\}, \eta) = \frac{1}{2} \sum_a \sum_{ij\alpha} \Lambda_{ai\alpha}^{(1,1)} \eta_a (u_{i\alpha} - u_{j\alpha}) + \frac{1}{6} \sum_a \sum_{ij\alpha kh\beta} \Lambda_{ai\alpha j\beta}^{(1,2)} \eta_a (u_{i\alpha} - u_{j\alpha})(u_{k\beta} - u_{h\beta}) + \dots$$

Fitting anhamonic terms

Fit is performed on both Phonon and Strain-phonon terms.

- For a set (TS) of configurations (s) from DFT calculations, we fit with least squares method, the goal function is defined as:

$$G[\lambda_p, TS] = \frac{1}{M_1} \sum_{s, \alpha, j} (F_{\alpha_j}^{TS}(s) - F_{\alpha_j}[\lambda_p](s))^2 + \frac{1}{M_2} \sum_{s, j} \Omega^2(s) (\sigma_j^{TS}(s) - \sigma_j[\lambda_p](s))^2$$

- where $\Omega(s) = (V(s)\sqrt{(N)})^{(-1/3)}$
- The goal function has to satisfy $\frac{\partial G[\lambda_p, TS]}{\partial \lambda_\mu} = 0 \quad \forall \mu$ and $\frac{\partial^2 G[\lambda_p, TS]}{\partial \lambda_\mu \partial \lambda_\nu} \geq 0 \quad \forall \mu \nu$
- We solve the system of p linear equations in order to get the set of coefficients λ_p

```
prt_model = 1

#-----
#Inputs for the fitted coefficients
#-----

fit_coeff = 1

fit_generateCoeff = 1
fit_rangePower = 3 4      # Range for the powers of the polynomial (default 3 to 4 )
fit_ncoeff = 7           # Number of coefficients to consider for the fit
fit_cutoff = 8           # Cutoff for the interactions
fit_anhaStrain = 0
fit_SPCoupling = 1

ts_option = 1
```

Problem: We create much more terms than we can fit !

Perovskite:

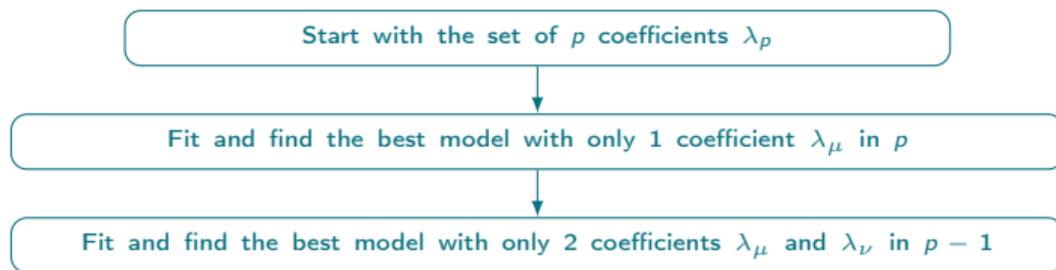
- 1st neighbor($0.5a$): 20-terms
- 2nd neighbor($a/\sqrt{2}$): 278-terms
- 3rd neighbor(a): 1504-terms

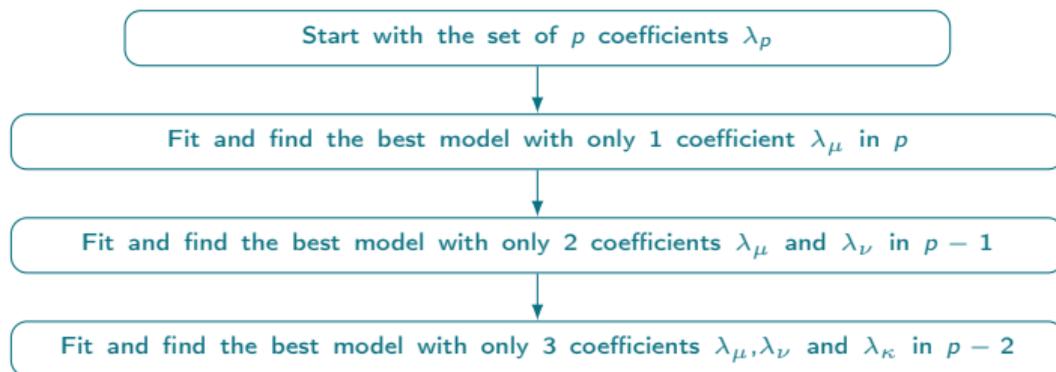
Start with the set of p coefficients λ_p

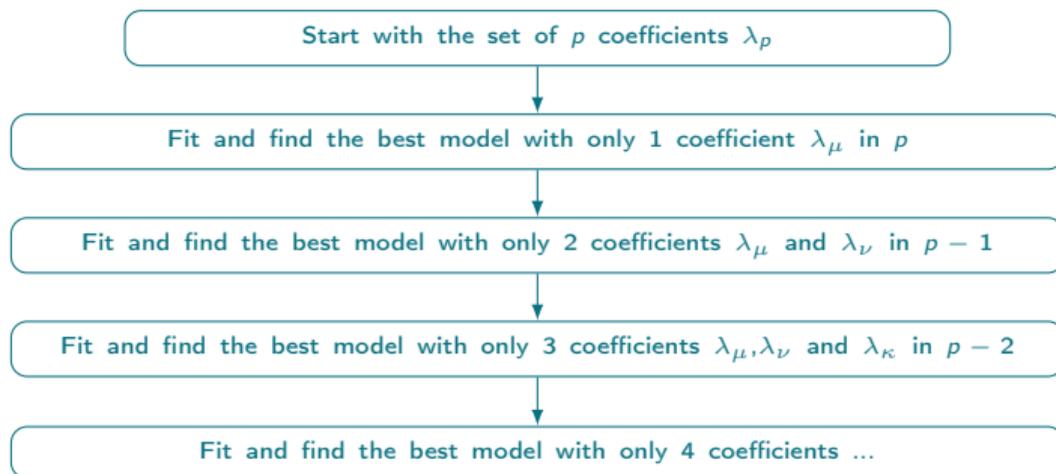
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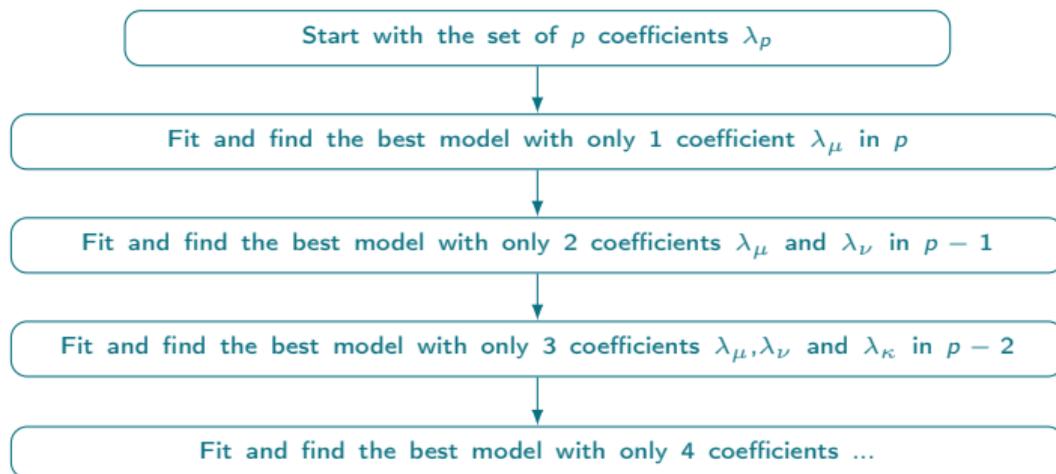


Fit and find the best model with only 1 coefficient λ_{μ} in p

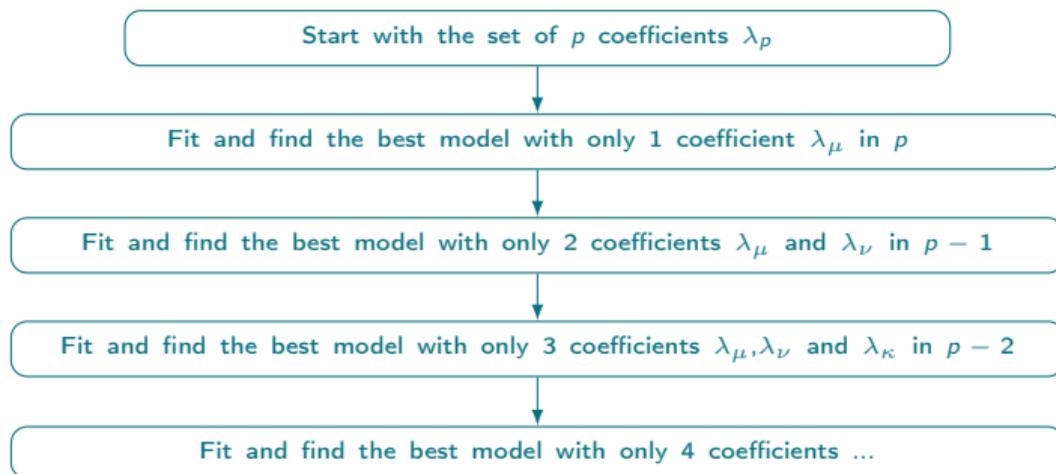






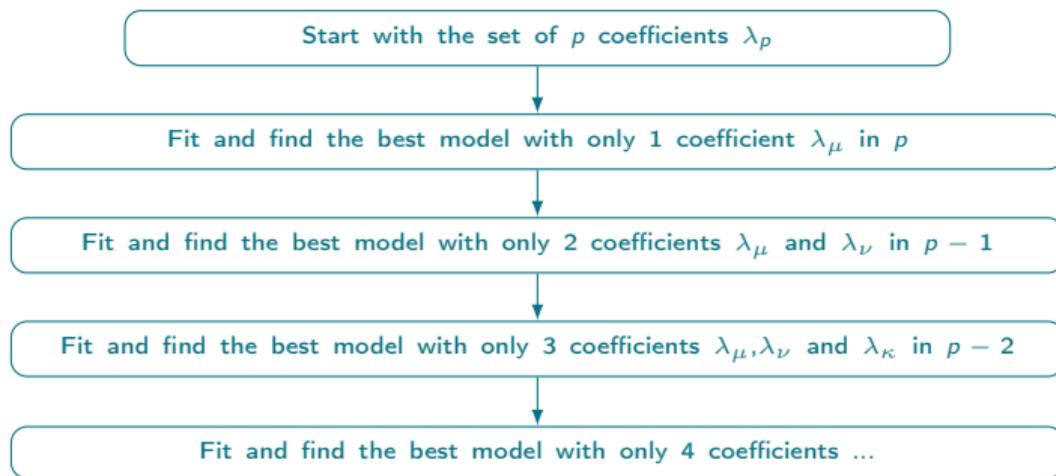


Problems:



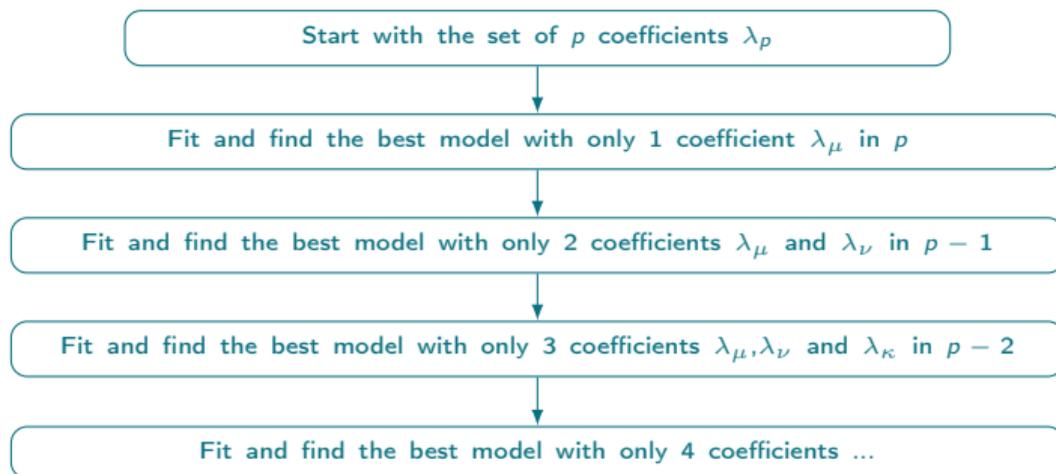
Problems:

- We need to define convergence criterion to stop the fit process (specific phase, energy, forces...)



Problems:

- We need to define convergence criterion to stop the fit process (specific phase, energy, forces...)
- We need to define a good training set

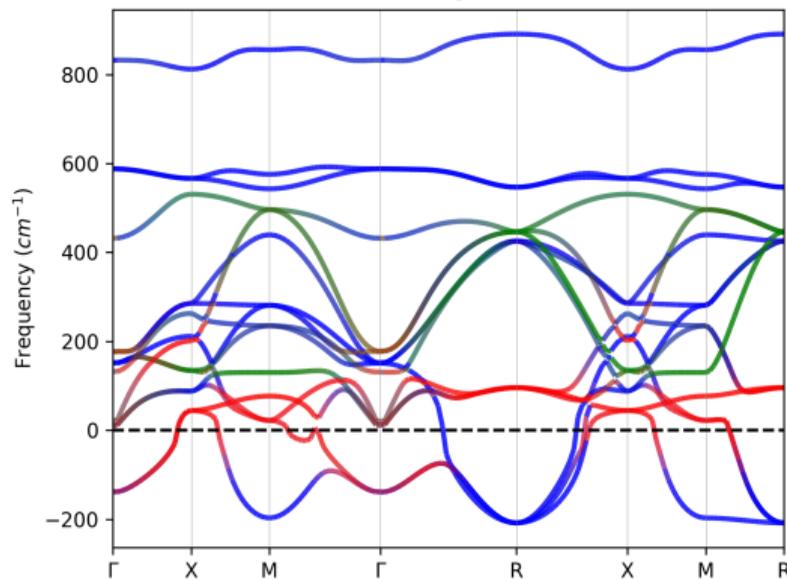


Problems:

- We need to define convergence criterion to stop the fit process (specific phase, energy, forces...)
- We need to define a good training set
- We need to bound the model

Real Materials Might Have Many Instabilities and Complex Ground-State Structures

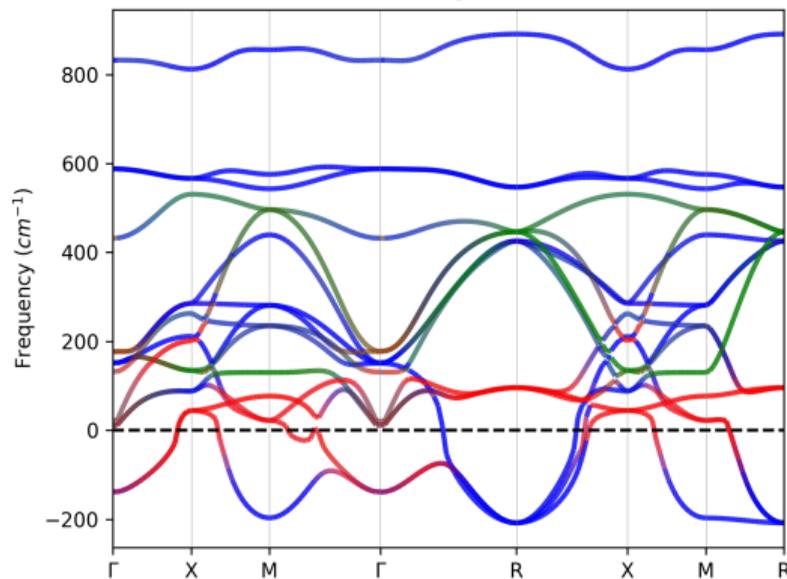
CaTiO_3



Ground-State Structure $Pnma$: Composed of 5 modes and 2 strains

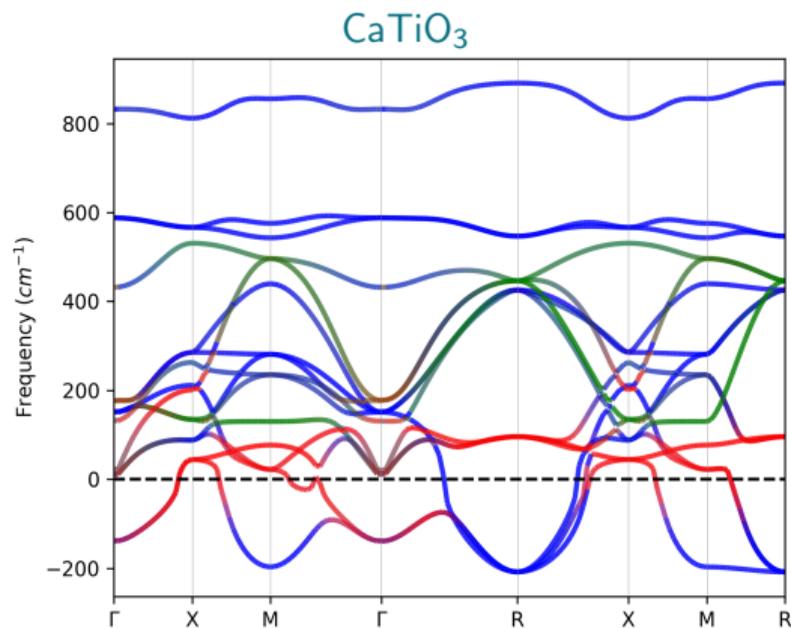
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Largest displacement about $0.4 \text{ \AA} \approx 10\%$ of LC

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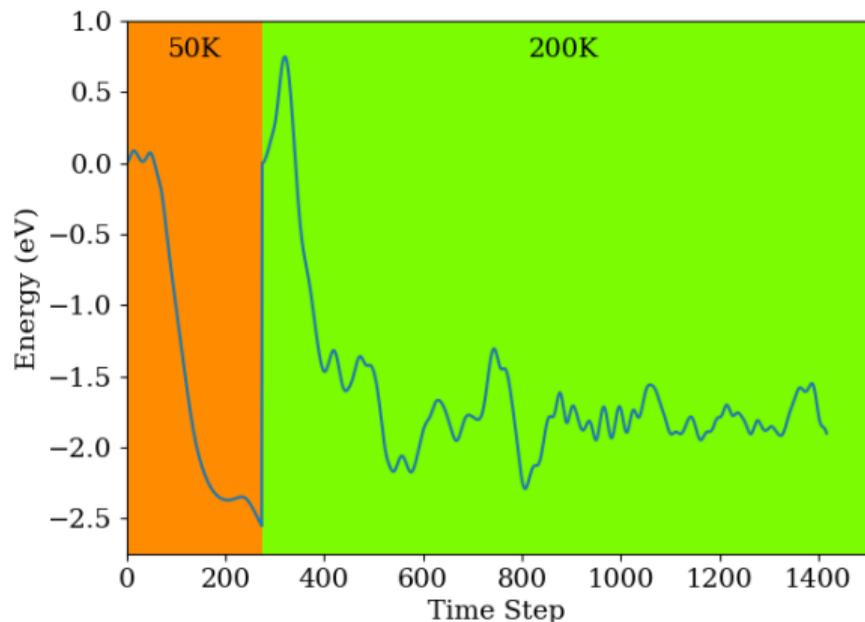


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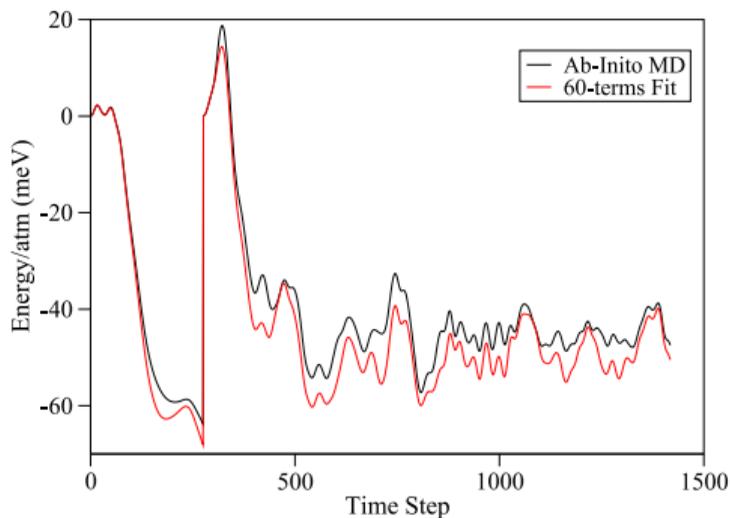
Largest displacement about $0.4 \text{ \AA} \approx 10\%$ of LC

Competing Phases $R\bar{3}c, I4/mcm, P4/mbm, Cmcm$

Use Ab-Initio Molecular Dynamics to Sample Instable Paths



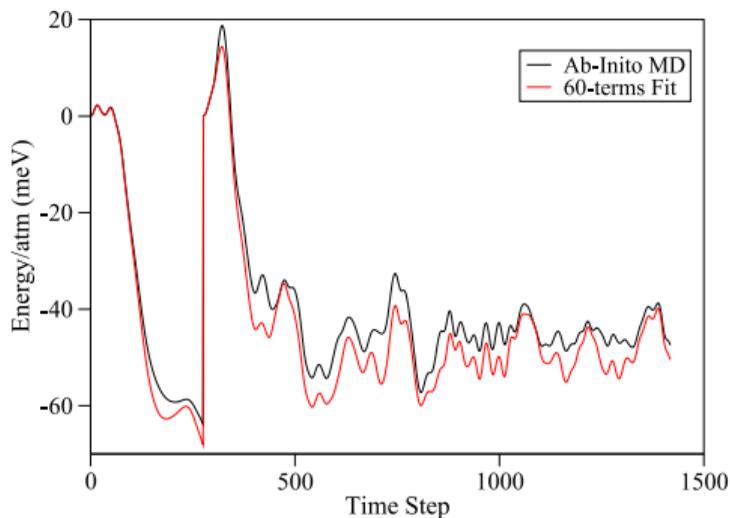
A First Free Fit Using Multibinit



Mean Standard Deviation values of the effective-potential (meV/atm):

```
Energy      : 4.0803665397763584E+00
Goal function values of the effective-potential
with respect to the test-set (eV^2/A^2):
Forces+Stresses : 2.4466967954928562E-02
Forces         : 2.0260908611633852E-02
Stresses       : 4.2060593432947067E-03
```

A First Free Fit Using Multibinit

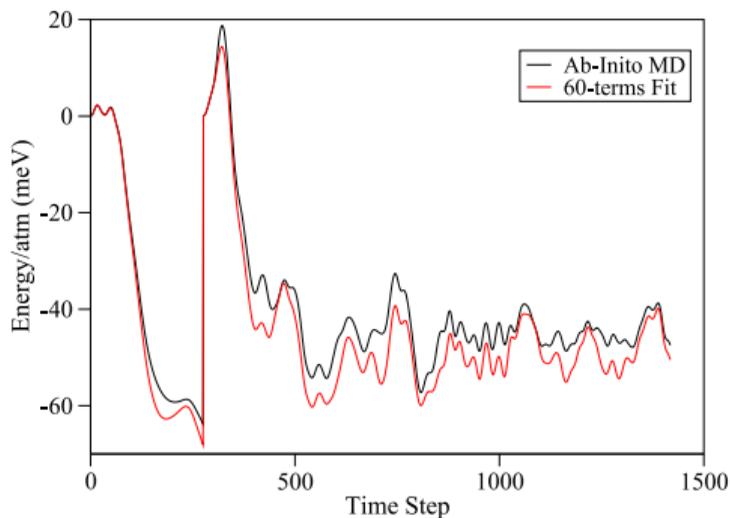


At $T=50\text{K}$ Model goes
to the $Pnma$ -phase

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```

A First Free Fit Using Multibinit



At $T=50\text{K}$ Model goes
to the *Pnma*-phase

5000-steps, 40-atoms
 $\approx 70\text{s}$ on 4 cores

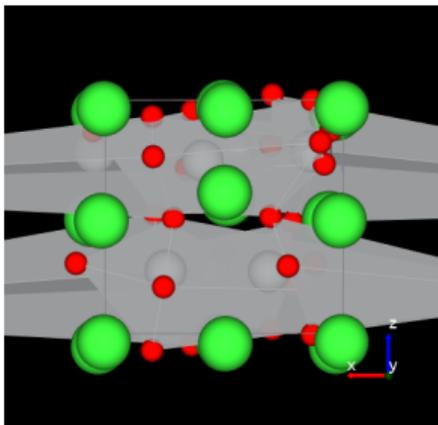
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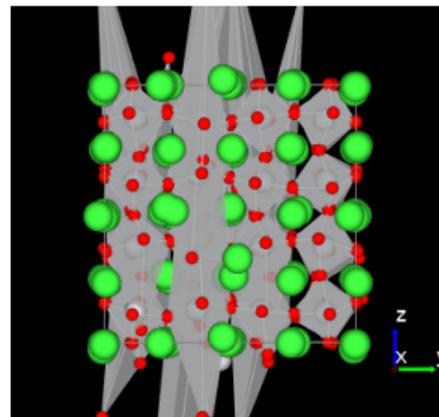
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```

"Boundedness" is a Big Problem

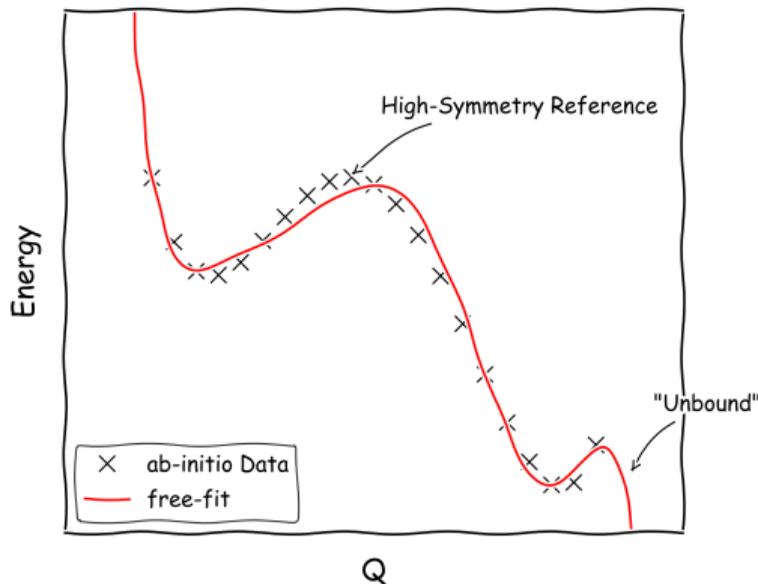
$T = 300\text{K}$



$\text{ncell} = 4 \times 4 \times 4$

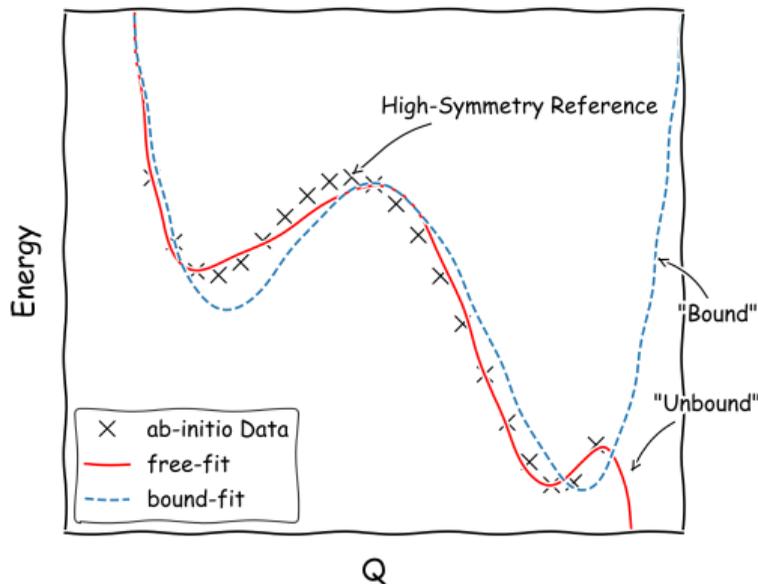


"UnBoundedness" - Negative Divergence in The Effective Potential



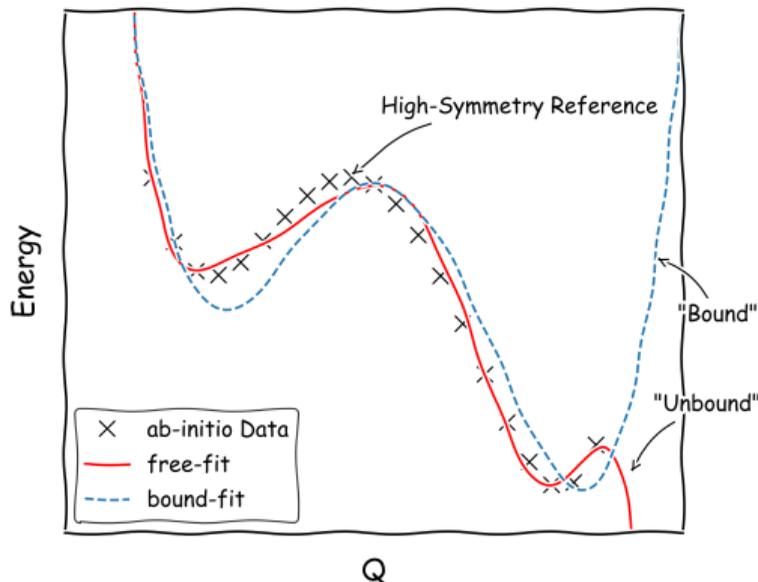
Appears if highest order term in Q is odd or even with negative coefficient

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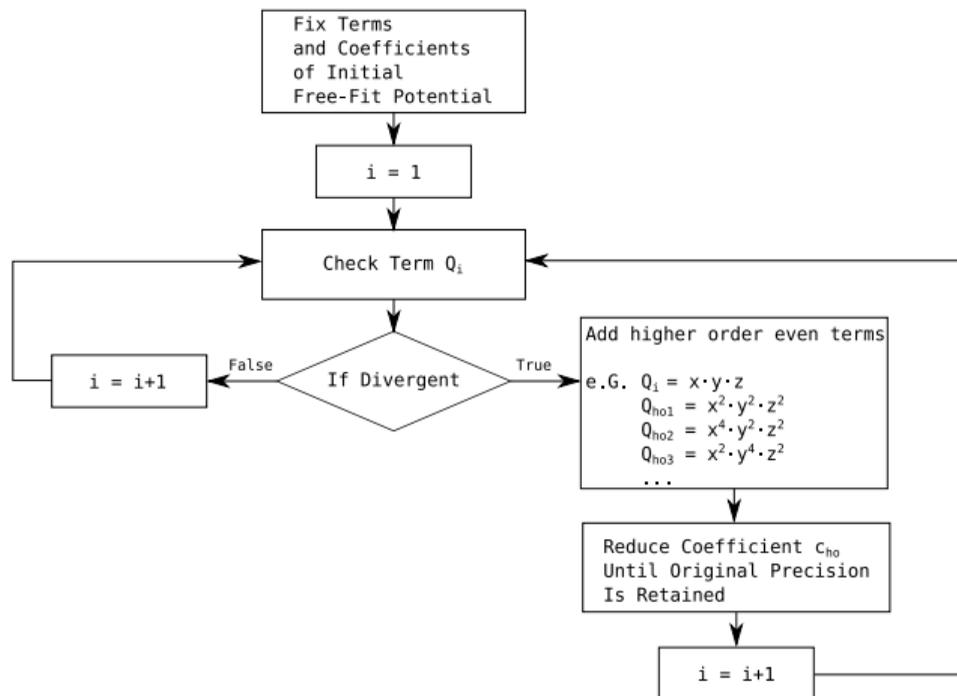
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"UnBoundedness" - Negative Divergence in The Effective Potential



Appears if highest order term in Q is odd or even with negative coefficient
Add higher order terms to bound in direction Q ! How to keep precision?

A Simple Algorithm to Impose Boundedness



Free Fit
 60 terms

```

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```

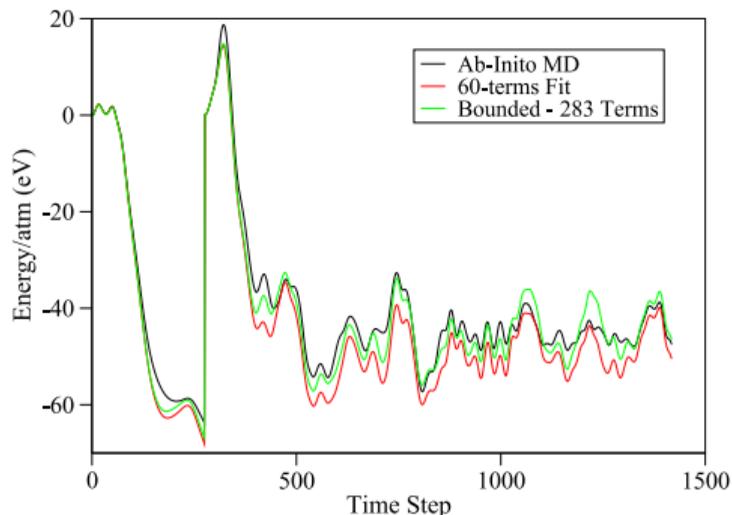


Mean Standard Deviation values of the effective-potential (meV/atm):

Energy	:	4.0803665397763584E+00
--------	---	------------------------

Goal function values of the effective-potential
with respect to the test-set ($\text{eV}^2/\text{\AA}^2$):

Forces+Stresses	:	2.4466967954928562E-02
Forces	:	2.0260908611633852E-02
Stresses	:	4.2060593432947067E-03



Mean Standard Deviation values of the effective-potential (meV/atm):

Energy : 2.1664871069774949E+00

Goal function values of the effective-potential
with respect to the test-set ($\text{eV}^2/\text{\AA}^2$):

Forces+Stresses : 2.7889442532740654E-02

Forces : 2.3225875500530173E-02

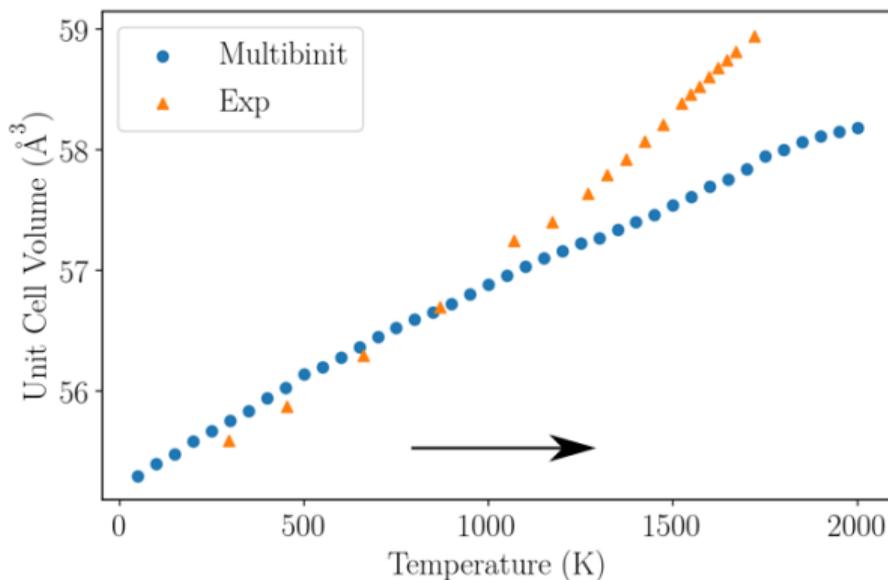
Stresses : 4.6635670322104803E-03

The model is bound

16x16x16 cells, 20480 atoms

6000-steps per temperature

96-cores \approx 1h15min per temperature



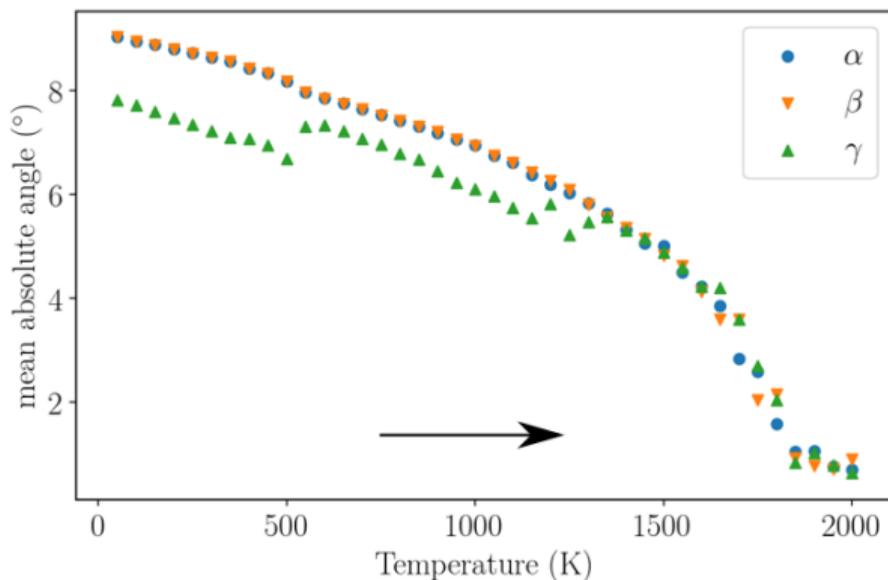
Yashima, M. & Ali, R., Solid State Ionics, 2009, **180**, 120 - 126

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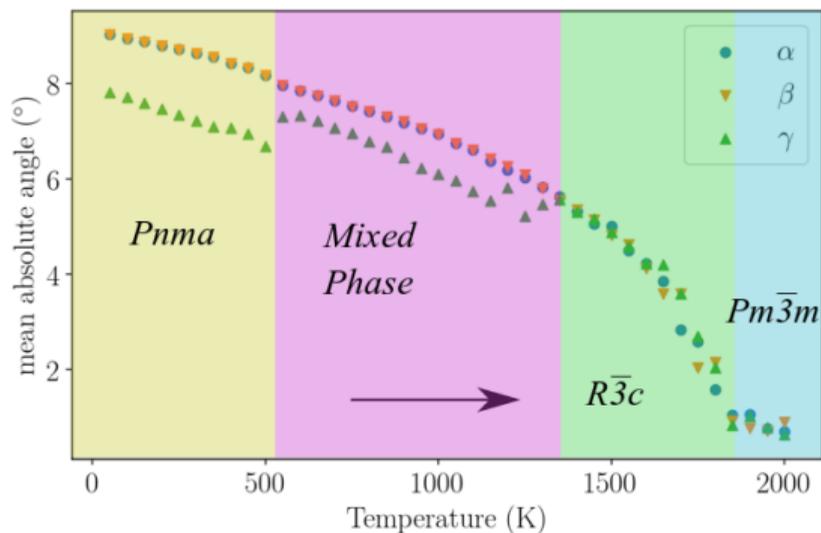
Yashima, M. & Ali, R., *Solid State Ionics*, 2009, **180**, 120 - 126

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Experimental Transitions: $Pnma \xrightarrow{1512K} I4/mcm \xrightarrow{1636K} Pm\bar{3}m$

Yashima, M. & Ali, R., Solid State Ionics, 2009, 180, 120 - 126

The Information about the electronic states are hidden in the lattice effective potential parameters

$$E_{tot}(\mathbf{u}, \eta) = E_0(\mathbf{r}_0, 0) + E(\mathbf{u}, \eta)$$

$$E(\mathbf{u}, \eta) = E^{phonon}(\mathbf{u}) + E^{strain}(\eta) + E^{strain-phonon}(\mathbf{u}, \eta)$$

$$E^{ph}(\mathbf{u}) = \sum_{ijkh\alpha\beta} K_{ijkh\alpha\beta}^{(2)} (u_{i\alpha} - u_{j\alpha})(u_{k\beta} - u_{h\beta}) + \sum_{ikhrt\alpha\beta\gamma} K_{ikhrt\alpha\beta\gamma}^{(3)} (u_{i\alpha} - u_{j\alpha}) \times (u_{k\beta} - u_{h\beta})(u_{r\gamma} - u_{t\gamma}) \dots$$

$$E^s(\eta) = \sum_{ab} C_{ab} \eta_a \eta_b$$

$$E^{s-ph}(\{\mathbf{u}\}, \eta) = \sum_a \sum_{ij\alpha} \Lambda_{aj\alpha}^{(1,1)} \eta_a (u_{i\alpha} - u_{j\alpha}) + \sum_a \sum_{ijkh\alpha\beta} \Lambda_{ajhk\alpha\beta}^{(1,2)} \eta_a (u_{i\alpha} - u_{j\alpha}) \times (u_{k\beta} - u_{h\beta}) \dots$$

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Reintroduce some electronic states of interest with SCALE-UP

Garcia-Fernandez, P. et al Phys. Rev. B, 2016, 93 195137

$$E_{tot}(\mathbf{u}, \eta) = E_0(\mathbf{r}_0, 0) + E(\mathbf{u}, \eta) + E_{el}(\mathbf{u}, \eta)$$

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With the central quantity: $D_{ab} = d_{ab} - d_{ab}^{(0)}$

6. Electronic + Lattice Effective Model

The Information about the electronic states are hidden in the lattice effective potential parameters

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Garcia-Fernandez, P. et al Phys. Rev. B, 2016, 93 195137

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With the central quantity: $D_{ab} = d_{ab} - d_{ab}^{(0)}$

And the electron-lattice coupling expressed in $\gamma_{ab}(\mathbf{u}, \eta)$

The Information about the electronic states are hidden in the lattice effective

Access

- Electronic Structure at Finite T
- Electronic Structure of Large Scale Objects
- $D_{ab} \neq 0$: Magnetic States, Polarons, Excitons

$E^{ph}(\mathbf{u}) =$
+
x

Reintroduce some electronic states of interest with SCALE-UP

Garcia-Fernandez, P. et al Phys. Rev. B, 2016, 93 195137

$$E_{tot}(\mathbf{u}, \eta) = E_0(\mathbf{r}_0, 0) + E(\mathbf{u}, \eta) + E_{el}(\mathbf{u}, \eta)$$

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With the central quantity: $D_{ab} = d_{ab} - d_{ab}^{(0)}$

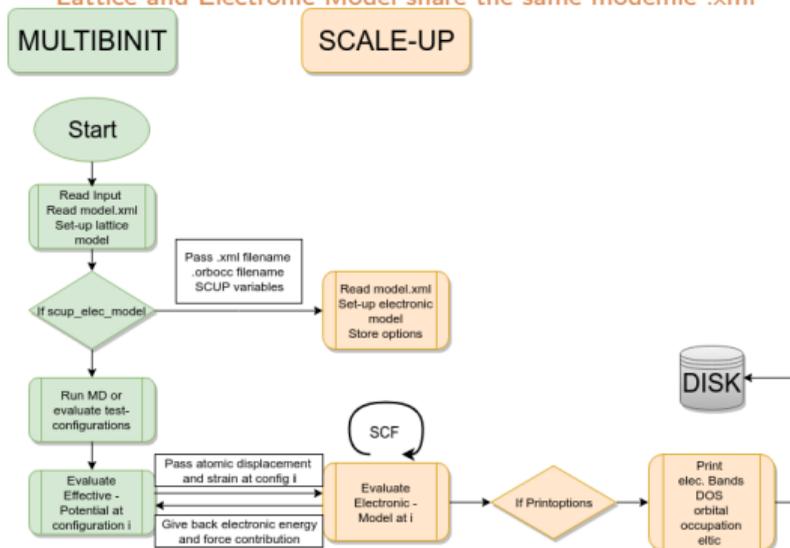
And the electron-lattice coupling expressed in $\gamma_{ab}(\mathbf{u}, \eta)$

THE MULTIBINIT - SCALE-UP INTERFACE

Multibinit incorporates SCALE-UP as a library

```
FC_LIBS="-L/path/to/scaleup/build/src/.libs/ -lscaleup"
```

Lattice and Electronic Model share the same modelfile .xml



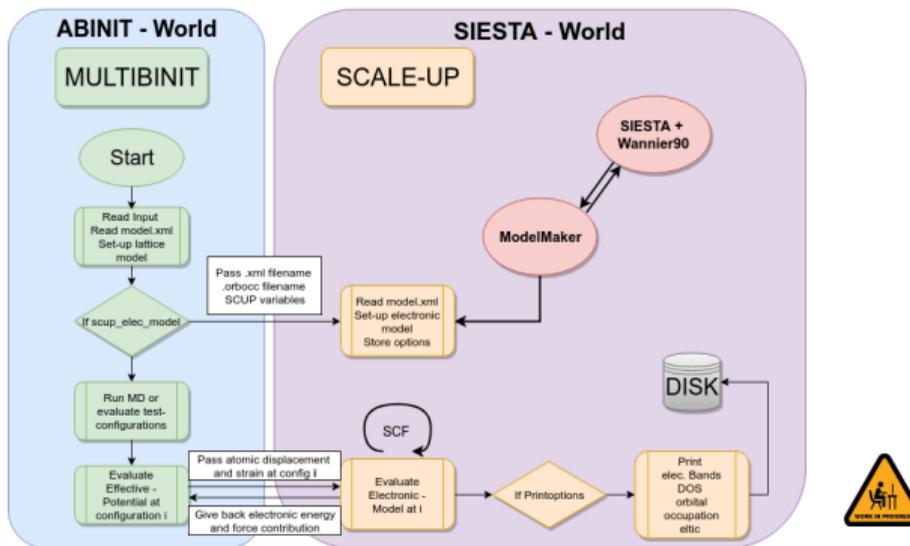
6. Electronic + Lattice Effective Model

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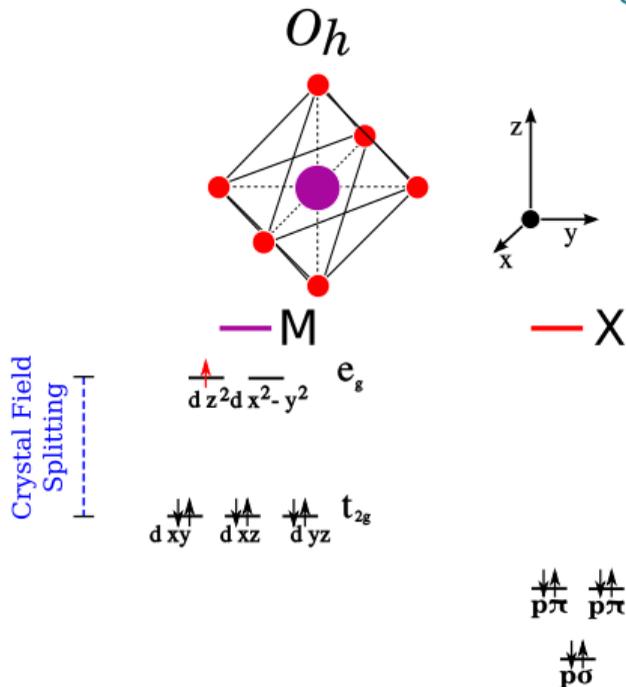
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Transferability assured by PSML Pseudopotential-Format.

A first model using electron-lattice coupling

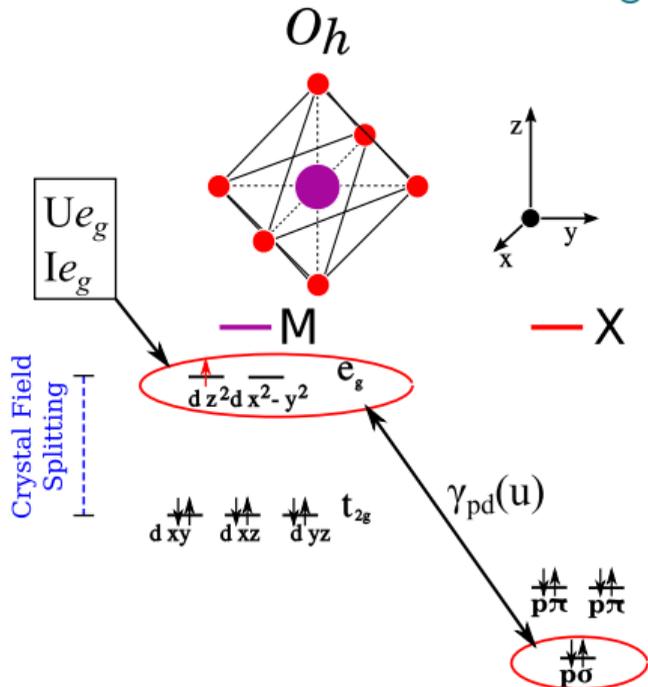
Classical Problem of cooperative Jahn-Teller Effect in Perovskites
Corner shared transition metal octahedra with double-degenerate electronic state



6. Electronic + Lattice Effective Model

A first model using electron-lattice coupling

Classical Problem of cooperative Jahn-Teller Effect in Perovskites
Corner shared transition metal octahedra with double-degenerate electronic state

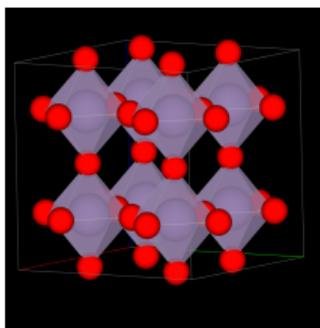
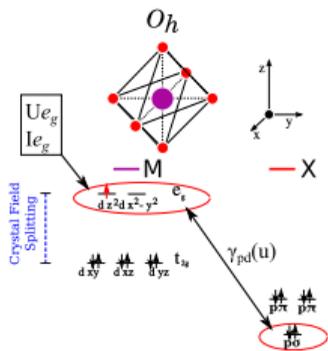


6. Electronic + Lattice Effective Model

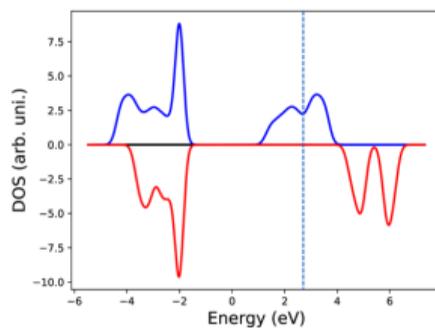
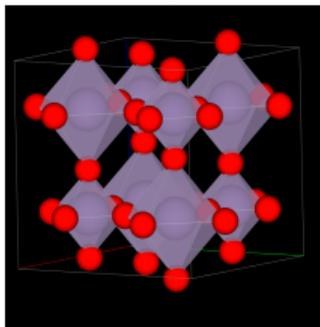
Electron-Lattice Coupling - Investigate Band-Structure and DOS

$$U_{eg} = 3\text{eV}, I_{eg} = 1.5\text{eV}, \gamma_{pd} = 1.5\text{eV}$$

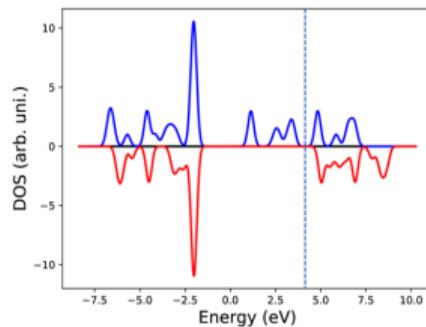
Cubic



Q_1^R



↑ Spin-up



↓ Spin-Down

Multibinit is



But first interesting results arrive !

Thank You !

- ULiège : Marcus Schmitt, He Xu, Nicole Helbig, Fabio Ricci, Matthieu Verstraete, Eric Bousquet, Philippe Ghosez
- UCLouvain : Gian-Marco Rignanese, Xavier Gonze
- Others : Alexandre Martin, Sergeï Prokhorenko, Andrés Camilo García Castro

Interactions also with :

- USantander : Javier Junquera and Pablo García-Fernández
- LIST : Jorge Íñiguez